Paper Reference(s)

# 6663/01

# Edexcel GCE

## Core Mathematics C1

## Advanced Subsidiary

# Differentiation:

# Stationary points

**Calculators may NOT be used for these questions.**

### Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ might be needed for some questions.

The marks for the parts of questions are shown in round brackets, e.g. **(2)**.

There are 3 questions in this test.

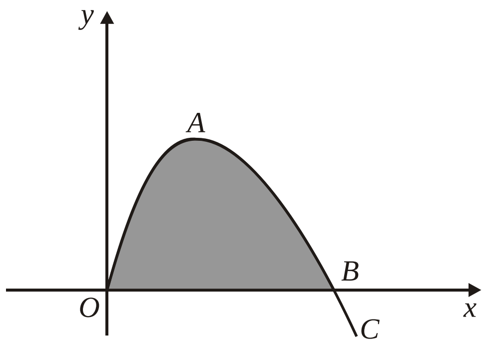
### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear.

Answers without working may not gain full credit.

**1.**




The figure above shows part of the curve $C$ with equation

$$y = 3x^{\frac{1}{2}} - x^{\frac{3}{2}}, \; x \geq 0.$$

The point $A$ on $C$ is a stationary point and $C$ cuts the $x$-axis at the point $B$.

(a) Show that the $x$-coordinate of $B$ is 3.

**(1)**

(b) Find the coordinates of $A$.

**(5)**

(c) Find the exact area of the finite region enclosed by $C$ and the $x$-axis, shown shaded in the figure above.

**(5)**

**(Total 11 marks)**

**2.** For the curve $C$ with equation $y = x^4 - 8x^2 + 3$,

(a) find $\frac{dy}{dx}$,

**(2)**

(b) find the coordinates of each of the stationary points,

**(5)**

(c) determine the nature of each stationary point.

**(3)**

The point $A$, on the curve $C$, has $x$-coordinate 1.

(d) Find an equation for the normal to $C$ at $A$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(5)**

**(Total 15 marks)**

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**3.** On a journey, the average speed of a car is $v$ m s$^{-1}$. For $v \geq 5$, the cost per kilometre, $C$ pence, of the journey is modelled by

$$C = \frac{160}{v} + \frac{v^2}{100}.$$

Using this model,

(a) show, by calculus, that there is a value of $v$ for which $C$ has a stationary value, and find this value of $v$.

**(5)**

(b) Justify that this value of $v$ gives a minimum value of $C$.

**(2)**

(c) Find the minimum value of $C$ and hence find the minimum cost of a 250 km car journey.

**(3)**

**(Total 10 marks)**

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|---|---|---|---|---|
| **1.** (a) | $y = 0 \;\Rightarrow\; x^{\frac{1}{2}}(3 - x) = 0 \;\Rightarrow\; x = 3$ | | B1 | 1 |
| | or $3\sqrt{3} - 3^{\frac{3}{2}} = 3\sqrt{3} - 3\sqrt{3} = 0$ | | | |
| (b) | $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$ | $x^n \mapsto x^{n-1}$ | M1 A1 | |
| | $\frac{dy}{dx} = 0 \Rightarrow x^{\frac{1}{2}} = x^{-\frac{1}{2}}$ | Use of $\frac{dy}{dx} = 0$ | M1 | |
| | $\Rightarrow x = 1$ | | A1 | |
| | $A$: (1, 2) | | A1 | 5 |
| (c) | $\int\left(3x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)dx = 2x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}$ | M1 $x^n \mapsto x^{n+1}$ | M1 A1+A1 | |
| | Accept unsimplified expressions for As | | | |
| | Area = $[...]_0^3 = 2 \times 3\sqrt{3} - \frac{2}{5} \times 9\sqrt{3}$ | Use of correct limits | M1 | |
| | Area is $\frac{12}{5}\sqrt{3}$ (units$^2$) | | A1 | 5 |
| | For final A1, terms must be collected together but accept exact equivalents, e.g. $\frac{4}{5}\sqrt{27}$ | | | |

**[11]**

**2.** (a) $\frac{dy}{dx} = 4x^3 - 16x$ M1 A1 2

(b) $4x^3 - 16x = 0$ M1

$4x(x^2 - 4) = 0$ $x = 0,\ 2,\ -2$ A2 (1, 0)

$y = 3,\ -13,\ -13$ M1 A1 5

(c) $\frac{d^2y}{dx^2} = 12x^2 - 16$ M1

$x = 0$ Max.

$x = 2$ Min. One of these A1 ft

$x = -2$ Min. All three A1 3

(d) $x = 1$: $y = 1 - 8 + 3 = -4$ B1

At $x = 1$, $\frac{dy}{dx} = 4 - 16 = -12\ (m)$ B1 ft

Gradient of normal $= -\frac{1}{m}$ $\left(= \frac{1}{12}\right)$ M1

$y - (-4) = \frac{1}{12}(x - 1)$ $x - 12y - 49 = 0$ M1 A1 5

**[15]**

**3.** (a) $\frac{dC}{dv} = -160v^{-2} + \frac{2v}{100}$ M1 A1

$-160v^{-2} + \frac{2v}{100} = 0$ M1

$v^3 = 8\,000$ $v = 20$ M1 A1

(b) $\frac{d^2C}{dv^2} = 320v^{-3} + \frac{1}{50}$ M1

$> 0$, therefore minimum A1

(c) $v = 20 : C = \frac{160}{20} + \frac{400}{100} = 12$ B1 ft

Cost = 250 × 12 = £30 M1 A1

**[10]**

**1.** The first part of this question gave difficulty to many. There are a number of possible approaches but many just wrote down $3x^{\frac{1}{2}} - x^{\frac{3}{2}} = 3\sqrt{3} - 3^{\frac{3}{2}} = 0$ and this was thought inadequate unless they could show that $3^{\frac{3}{2}}$ or $\sqrt{27}$ was $3\sqrt{3}.3^{\frac{3}{2}} = 3 \times 3^{\frac{1}{2}}$ would have been sufficient demonstration of this. In part (b), not all could solve $\frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0$ and a few found the second derivative and equated that to zero. In part (c), most could gain the first four of the five marks available but cleaning up the final answer to the single surd, $\frac{12}{5}\sqrt{3}$ or its equivalent, proved difficult and many had recourse to their calculators, which did not fulfil the condition of the question that an exact answer is to be given.

**2.** The differentiation in part (a) was completed successfully by nearly all candidates, but some made no further progress, perhaps not understanding the meaning of “stationary point”. Most, however, knew in part (b) that they needed to use $\frac{dy}{dx} = 0$, but a very common mistake in solving $4x^3 - 16x = 0$ was to omit one or two of the three possible solutions. Some candidates also failed to find the corresponding *y* coordinates. Use of the second derivative in part (c) was the preferred method to determine the nature of the stationary points, and some good solutions were seen here, although some candidates tried equating their second derivative to zero either in this part or part (b). Finding the equation of the normal in part (d) proved difficult for some candidates. While most found the *y*-coordinate of *C*, some did not realise that they needed to use $\frac{dy}{dx}$ to find the gradient, then others found the equation of the tangent instead of the normal. Where methods *were* correct, algebraic or arithmetic slips often led to the loss of at least one mark, but the correct answer in the required form was not uncommon.

**3.** Many candidates found this question difficult, producing incomplete solutions. It was clear in part (a) that some did not understand the meaning of “stationary value”, and despite the reference to calculus, a great deal of irrelevant algebra was seen. Sometimes the given formula was multiplied by 100*v* before a differentiation attempt, and sometimes candidates thought that they needed to differentiate separately the 100 denominator to give, for example, $-100^{-2}$. Dealing with $160v^{-1}$ was sometimes a problem, but those who managed to differentiate *C* correctly often proceeded to solve the resulting equation correctly.

In part (b), the method of justifying the minimum by considering the value of the second derivative was usually well known, and follow-through credit was given here where possible. Answers to part (c) were sometimes clear and concise, but sometimes very confused, with $v = 5$ or other inappropriate values being used to find the minimum value of *C*. Some candidates did not continue to find the minimum cost of a 250 km car journey, while others seemed to be confusing 250 km with 250 km $h^{-1}$.